Introduction to Elementary Particle Physics

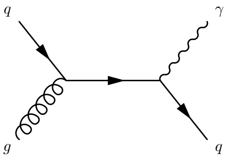
2: An overview of Calculations

Dr. Sahal Yacoob

18 August 2020

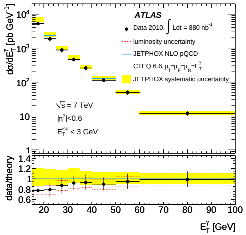
Observable Quantities

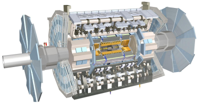
2To test a theory, it must predict something measurable.



Theory

!





Experiment

The *Second Golden Rule* with the *Feynman Rules* can predict *decay rates* and *cross sections*

- *Decay Rate:* The probability per unit time that a particle will disintegrate.
- *Cross Section:* The 'area' of interaction between particles.

Cross Section

The *cross section* ( ) gives the 'effective area' of an interaction. It is proportional to the probability of the interaction to have a particular outcome.

The dimension of   is area, commonly used unit is the *barn (b)*[1]:

$$1\,b = 10^{\;\;28}\,m^2$$

Note that an individual particle does not have a cross section. The *interaction* between two (or more) particles has a cross section!

[1]comes from the expression \You couldn't hit the broad side of a barn".

Di erential Cross Section

In an interaction, particles are scattered into di erent directions.

$\frac{d}{d}$ is the di erential cross section, b is called the impact parameter.

$$d \;=\; |b\,db\,d\,| \text{ and } d \;=\; |\sin \; d \; d \;| \;\rightarrow\; \frac{d}{d} = \frac{b}{\sin}\;\frac{db}{d}$$

$$\frac{d}{d} = \frac{\text{Number of interactions per target particle that lead to scattering into } d \text{ at angle}}{(\text{Number of incident particles per unit area})}$$

Cross Section of a Hard Sphere

Di erential cross section:

$$b = R\cos(= 2)$$

$$\frac{db}{d} = \frac{R}{2}\sin \frac{}{2}$$

$$\frac{d}{d} = \frac{Rb\sin(= 2)}{2\sin} = \frac{R^2}{4}$$

Total cross section:

$$= \int d = \int \frac{d}{d} d$$

$$= \int \frac{R^2}{4} d = R^2$$

Cross Section & Luminosity

Luminosity (L) is the number of particles passing (or possible interactions) per unit area per unit time [$cm^{\ 2}s^{\ 1}$].

The number of particles scattered (N_{scat}) per unit time from target of size   is then:

$$\frac{dN_{scat}}{dt} = \quad L$$

Z

$$N_{scat} = \qquad L \;\, dt$$

If the LHC has collected $25 fb^{\ 1}$ of data, how many proton-proton collisions have they produced? ($_{pp}$ '   100mb)

Decay Rates

Most particles decay (! $e^+ e$ [2] and ! e $_e$)

Particles have no memory. The probability of a muon decaying in the next microsecond is independent of when the muon was created. So, the decay rate ( ) is de ned by:

$$N(t) = N(0)\, e^{\quad t}$$

Most particles decay in several ways (! $e^+ e$ or ! $^+$)

The total decay rate is the sum of individual decay rates:

$${}_{tot} = \sum_{i=1}^{n} {}_{i}$$

[2] is also called J=

Lifetime and Branching Ratio

The mean lifetime( ), often just called 'lifetime', is:

$$= \frac{1}{\text{tot}}$$

Lifetime is the time in the particle's rest frame.

The branching ratio is the fraction that decay to a particular mode.

$$\text{Br}(i) = \frac{i}{\text{tot}}$$

Resonances

Unstable particles form resonances with a Breit-Wigner shape:

$$\propto \frac{N}{(E - M)^2 + (\Gamma/2)^2}$$

The Second Golden Rule

The Second Golden Rule[3] (or Born approximation) is:

$$\underbrace{dW_{fi}}_{\text{interaction rate}} = 2 \underbrace{\left| \int d^3r \; {}_r V(r) \; {}_i \right|^2}_{\text{amplitude } (M)} \underbrace{}_{\text{phase space}}$$

$M \equiv \langle f | V | i \rangle$ contains the dynamical information of the interaction from state $|i\rangle$ to state $|f\rangle$ (potential (V), charge, spin, etc)

contains the kinematic information of the interaction $(p_A; p_B; p_C; p_D; \ldots)$

[3] the derivation is beyond the scope of this course

Golden Rule for Decay and Scattering

With the Born approximation, an assumption that we work with spin-averaged amplitudes, and a few pages of maths, the decay rate for a two-body decay ($A \to B + C$) can be shown to be:

$$= \frac{|p|}{8 m_A^2} |M|^2$$

Similarly, the differential cross section for a $2 \to 2$ scatter ($A + B \to C + D$) can be shown to be:

$$\frac{d}{d} = \left(\frac{1}{8}\right)^2 \frac{|M|^2}{(E_A + E_B)^2} \frac{|p_f|}{|p_i|}$$

$\mathcal{M}$ can be calculated with the Feynman Rules[4], so decay rates and cross sections can be calculated then compared to experiment.

$$\underbrace{\left(\frac{1}{8\pi}\right)^2 \frac{|\mathcal{M}|^2}{(E_A+E_B)^2}\frac{|p_f|}{|p_i|}}_{\text{theorist calculates}} = \frac{d\sigma}{d\Omega} = \underbrace{\frac{dN_{scat}}{d\Omega \int L\,dt}}_{\text{experimentalist measures}}$$

- yellow band is theory calculation
- black points are experimental data

[4] take honours particle physics if you want to see how

From Lagrangian to Particles & Interactions

RECALL...

Classical Mechanics

- de ne the Lagrangian
  $L = \frac{1}{2}mv^2 \quad V(x)$
- use the Euler-Lagrange equation
  $\frac{d}{dt} \frac{@L}{@\dot{x}} \quad \frac{@L}{@x} = 0$
- get the equation of motion
  $F = ma$
- you get Newton's Law

Quantum Field Theory

- de ne the Lagrangian (density)
  $L_f = i \quad @ \quad m$
- use the Euler-Lagrange equation
  $@ \quad \frac{@L}{@(@ \quad)} \quad \frac{@L}{@} = 0$
- get the `equation of motion'
  $(i \quad @ \quad m) \quad = 0$
- you get the Dirac Equation

The phase of the particle wave function is not observable. Therefore, the Lagrangian should be symmetric under local phase (ie. gauge) transformations. Is it?

Lagrangian Symmetry

Particle Interactions

Force is transmitted when a fermion emits or absorbs a boson:

electron absorbs photon

electron emits photon

electrons exchange photon

These are called Feynman diagrams

- time ows left to right
- arrow denotes particle (forward) or antiparticle (backward)
- the vertical axis has no physical meaning

Propagators

The (unobserved) particle exchanged is called the **propagator**.

Look at the first half of the $e^+e^- \to e^+e^-$ diagram:

Can you conserve 4-momentum (p) here?

Particles that are 'off mass shell' are called **virtual** particles.

- propagators are virtual
- initial and final state particles are **real**

A particle can be virtual provided doesn't live too long:

$$\Delta E \, \Delta t \geq \frac{1}{2}$$

Range of a Force

Take the general $AB \to AB$ interaction via particle X.

Look at lower vertex in A rest frame ($p_A^{initial} = 0$)

$$(m_A; 0) \to (E_A; p_A) + (E_X; p_X)$$

So,

$$
\begin{aligned}
\Delta E &= E_f - E_i \\
&= E_A + E_X - m_A \\
&= \sqrt{p_A^2 + m_A^2} + \sqrt{p_A^2 + m_X^2} - m_A
\end{aligned}
$$

The limit case $p_A \to 0$ gives $\Delta E = m_X$, so $\Delta E \geq m_X$

Therefore, Heisenberg says

$$\Delta E \, \Delta t \geq \frac{1}{2} \to \Delta t \approx \frac{1}{2\Delta E} \to t_{unmeasurable} = \frac{1}{2m_X} \to$$

Massive propagators have limited range, R (remember $c = \hbar = 1$).

Quantum Electrodynamics (QED)

Electromagnetism mediated by the photon and described by QED.
Every QED interaction is based on this vertex:

- the solid line (X) is any electromagnetically charged particle
- the squiggly line is a photon ()
- the coupling constant is $= \frac{1}{137}$

The vertex can be rotated to give other processes:

e scatter

e^+ e annihilation

e^+ scatter

e^+ e pair production

Some Examples of QED Interactions

$e \;\; ! \;\; e$	$e \;\; {}^{+} \; ! \;\; e \;\; {}^{+}$	${}^{+} \;\; ! \;\; dd$	$e \;\; ! \;\; e$

$e^{+} e \;\; ! \;\; e^{+} e$

$e \;\; e \;\; ! \;\; e \;\; e$

Higher Order Diagrams

The previous examples are the lowest order (LO) diagrams for the processes. Every process has higher order diagrams.

Next-to-leading order (NLO) diagrams for e ! e are:

Higher order diagrams are constructed by adding additional internal lines without adding external lines.

Note that each diagram is constructed of the fundamental QED vertex, each vertex with a `strength' proportional to.

Perturbation Theory

To calculate what happens in an interaction like e ! e ,
one must add the diagrams at every order:

+ +

$O(\ ^2)$ + $O(\ ^4)$ +

Because < 1, each higher order contributes a smaller amount to the result. Phew!

Weak Interactions

Weak interactions are mediated by W and Z

- the weak charge is rather complex...
- all fermions carry weak charge
- W boson couples charged leptons to neutrinos
- W boson can also change quark avour

There are 3 weak interaction vertices:

Weak Interaction Examples

e + ! e +

d + $_e$! u + e

n ! p + e + $_e$

! p +

Quantum Chromodynamics (QCD)

QCD describes the strong interaction mediated by the gluon

- the charge of the strong interaction is colour
- colour comes in 3 types: red, green, blue (plus anti-colours)
- only quarks and gluons carry colour charge

There are 3 fundamental QCD vertices:

The strong coupling constant is $_{s}$ & 1

Freedom and Con nement

The gluon carries colour, unlike the photon which does not carry electric charge, this has consequences...

Asymptotic Freedom:

- coupling constants: $_s$ & 1, while $<$ 1
- thankfully, at small distances, $_s$ becomes $<$ 1, so perturbation theory can be used for some QCD calculations
- this is called asymptotic freedom (quarks are \free to move around" inside a proton)

Con nement:

- no naturally occurring particles carry colour
- quarks are con ned to bound states with no net colour charge
- particles composed of quarks are called hadrons

Hadron Classi cation

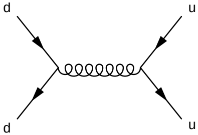
Hadron: a particle made of quarks is called a hadron

- Meson: a hadron made of a quark-antiquark pair
- Baryon: a hadron made of three quarks or three antiquarks

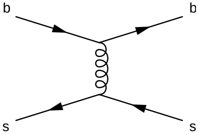
Examples:

	Quark Content	Spin	Charge	Mass (MeV)
Baryon				
p	uud	1=2	+1	938
p	u u d	1=2	-1	938
n	udd	1=2	0	939
^{0}	uds	1=2	0	1192
$^{+}$	uud	3=2	+1	1232
$^{++}$	uuu	3=2	+2	1232
Meson				
^{0}	(u u dd)= p 2	0	0	135
	u d, du	0	1	140
	u d, du	1	1	775
K	u s, su	0	1	494
D	cd, dc	0	1	1869
B	u b, bu	0	1	5279
	c c	1	0	3097
	bb	1	0	9460

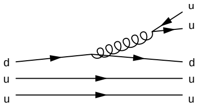
Example Diagrams



dd ! uu



bs ! bs



+ ! *p* + 0

Questions ?